



Teaching Math on an Oscilloscope, Part I: Integrals, Derivatives and FFT of a Sinusoid

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Purpose:

To show mathematical concepts in real time: integration, differentiation and FFT. This experiment is a way for the math, physics and engineering departments to work together to show students some “real” applications of math. It is useful in both high school and university environments.

Equipment:

- Agilent 33120A Synthesized Function/Arbitrary Waveform Generator
- Agilent 54600B 100 MHz Digital Oscilloscope with
- Agilent 54657A FFT and GPIB plug-on module

Prerequisites:

A very basic understanding of calculus and Fourier Transforms. If you want to simply show the fundamentals of a waveform before you begin this experiment, use the Measure keys on the scope and use various waveforms from the function generator to demonstrate voltage amplitudes (peak-to-peak, average and rms), time delays, frequency, period, duty cycle, pulse width, rise and fall times.

Theory:

The Agilent 54600B Digitizing Oscilloscope’s plug-on module has built-in math functions. The oscilloscope can perform these math functions in real time, and display the results on the oscilloscope screen for the student to see. The student can then play “what if” games to vary amplitude, dc offset and other parameters. We will begin with a simple sine wave and then set the DSO to “INTEGRATION”. The scope will display both the waveform and its integral (or derivative, or FFT). Keep in mind that the oscilloscope integrates EXACTLY what is seen on the screen. It performs the math function with the left edge of the screen assumed to be $t = 0$.

Agilent 33120A settings:

The oscilloscope input Z is very high. Normally, a function generator SHOULD be terminated in 50 ohms. If you have a 50 ohm termination, use it at the input terminal of the oscilloscope, and skip to the section marked ‘Set Up Waveform’. If you do not have a 50 ohm termination, set the function generator to expect the proper load impedance:

- SHIFT MENU
- >>> D: SYS MENU
- (down arrow) 1: OUT TERM
- (down arrow) > HI Z
- ENTER

The above settings will calibrate the amplitude readout on the Agilent 33120A for a high impedance load.



Set Up Waveform:

Set the AGILENT 33120A to generate a 1 kHz sine wave with 2 Vp-p amplitude:

- SINE WAVE
- AMPLITUDE; ENTER NUMBER; 2 [square wave key]; Vp-p [up arrow key]
- FREQUENCY; ENTER NUMBER; 1 [sine wave key]; kHz [down arrow key]

The function generator should now be generating a 1 kHz sine wave. Check it with the AGILENT 54600B DSO. Simply connect the Agilent 33120A output to CH1 of the scope and touch AUTOSCALE on the scope.

Set up Math Functions:

Turn on the scope's math function by hitting the +/- key, located between buttons CH1 and CH2; then:

- FUNCTION 2 ON (this is a softkey at the base of the screen)
- MENU
- Toggle the OPERATION key until the INTEGRAL function appears.
- You should see a waveform similar to FIG 1.

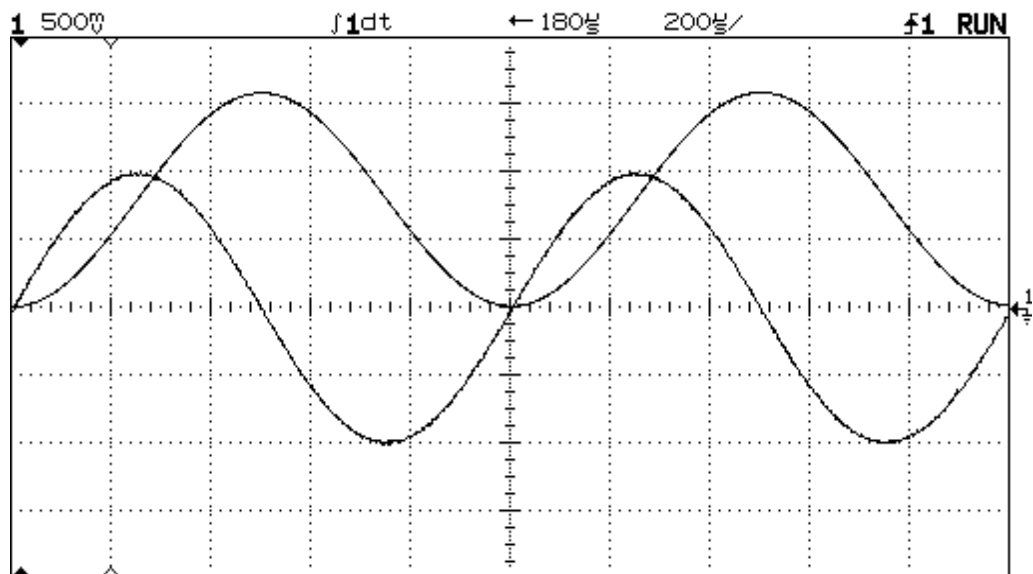


FIG 1: Integral of a sine wave.

Note from FIG 1 that the integral function is doing just what we expected. It is integrating the area under the sine wave curve. (Zero volts on the sine wave is indicated by the ground sign at the right edge of the screen.) [Note: If the integral waveform tends to ramp upward or downward, you may have a small offset on the function generator. This can be corrected one of two ways: Either set the scope coupling to AC [SLOPE/COUPLING; COUPLING AC], or with the scope in DC coupling, go to the function generator and set the offset so that the average value of the sine wave, V_{avg} , as measured with the automatic measurement function of the scope is close to zero. To do an automatic V_{avg} measurement, just touch Measure VOLTAGE, softkey V_{avg} .]

It is informative to change the DC offset on the function generator on purpose, to inject a slight dc offset and see the results on the integral. This simple exercise says a lot about source offsets, integration vs. initial conditions [hint: move the DSO's HORIZONTAL DELAY knob and watch the dc offset of the integral change.] This exercise should generate a lot of questions. For example, note what happens if we simply change the horizontal time scale and move the starting point by 1/2 cycle (See FIG 2):

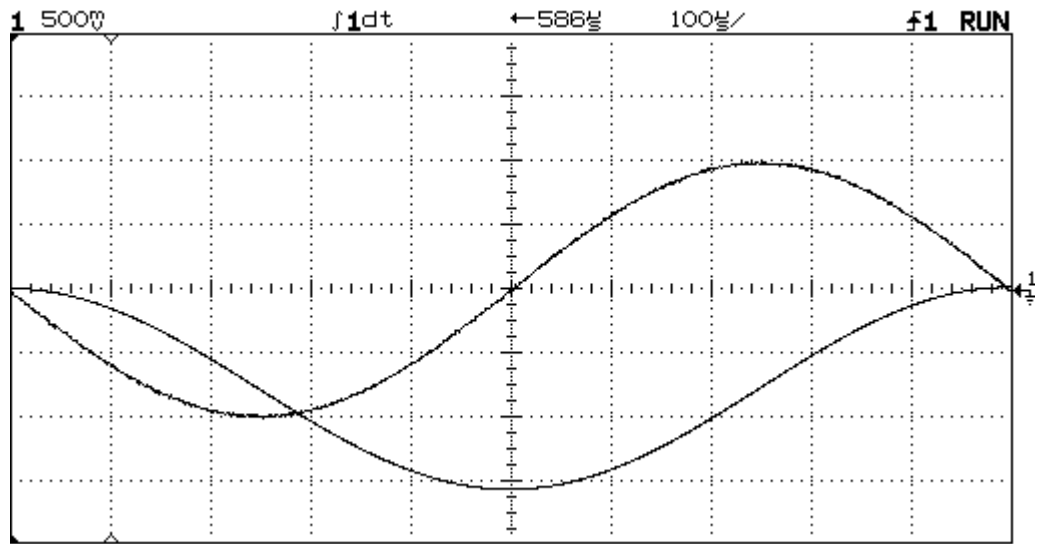


FIG 2: Integral of a negative sine wave.

Square Wave:

Change the DSO time base from its current setting of 100 microseconds/div to a new setting of 500 microseconds/division. Change the Agilent 33120A to SQUARE WAVE. You should see a waveform something like FIG 3.

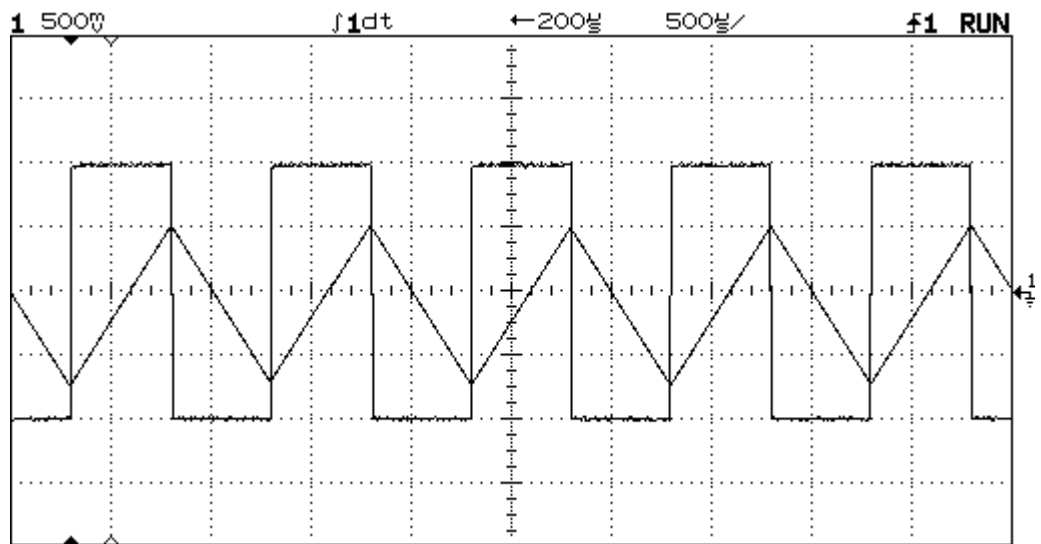


FIG 3: Integral of a square wave.

If you have the scope in DC COUPLING [SLOPE/COUPLING; DC] mode, and you either inject an offset using the Agilent 33120A OFFSET [OFFSET; move knob to desired setting] or if the Agilent 33120A has a slight offset inherent in its output, you will see the triangle wave ramping one way or the other, as in FIG 4.

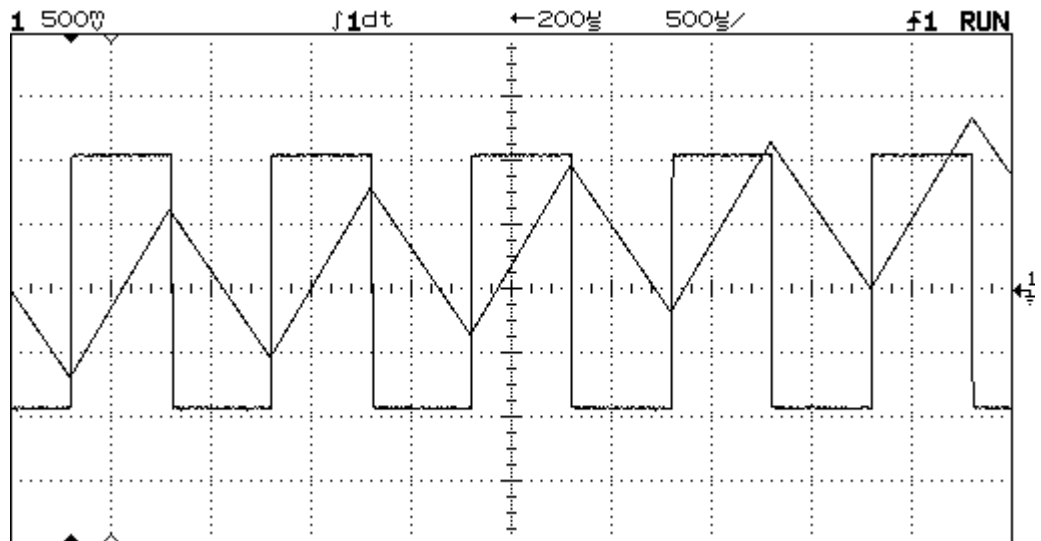


FIG 4: Ramped integral of a square wave with offset.

Why is the integral ramping? What happens if you... increase the offset? ... change its polarity?

.... Change from AC coupling to DC coupling? Try integrating other waveforms, such as triangle, ramp and noise. Explain the results.

Derivatives:

Now let's change to the derivative mode.

Return to SINE WAVE on the Agilent 33120A.

Set the scope to: +/- FUNCTION 2 MENU; Toggle the FUNCTION softkey until dV/dt appears.

With scope settings shown as in FIG 5, and the function generator set at a 1 kHz, 2 Vp-p sine wave, you should see something like FIG 5.

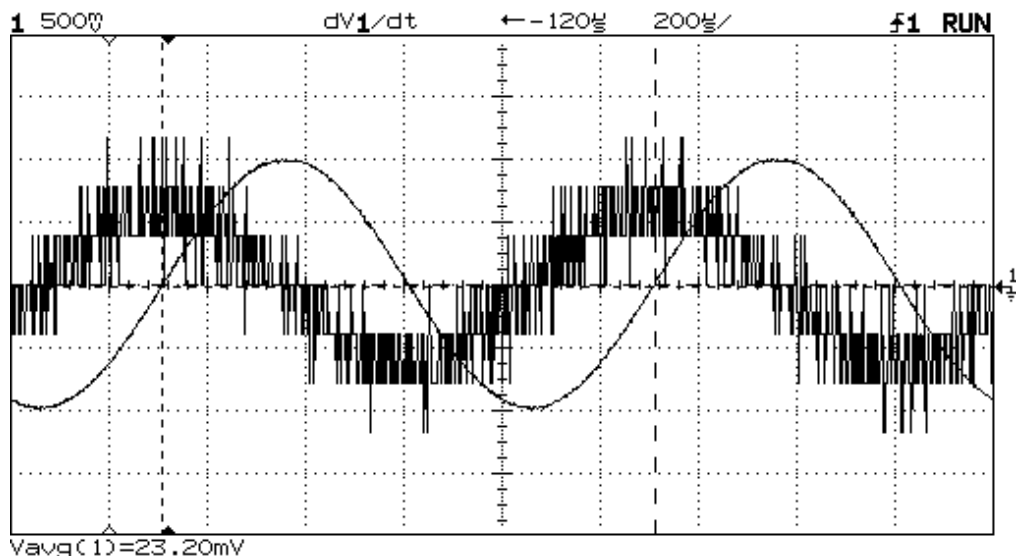


FIG 5: Derivative of sine wave.



Derivatives (continued)

What a mess! You can see the form of the derivative following the slope of the sine wave, but why all the noise?

The scope is indicating correctly. All that “noise” is really the derivative of the signal, caused by very minor disturbances in the signal... disturbances of one least-significant-bit. For example, if one lsb is worth $(500\text{mV/div} * 8 \text{ div})/256 = 4 \text{ volts}/256 = \text{about } 16 \text{ mv}$, and one point horizontally is worth about 4 microseconds, that means the derivative of just the tiniest voltage change that we can resolve will create a derivative of $16\text{mV}/ 4\text{us} = 4000\text{V/s}$. Derivatives cause a LOT of noise.

You can try other waveforms and settings, but what this all means is that derivative circuits are messy, prone to noise and rf pickup, and should be avoided wherever possible.

Fourier Transforms

FFT of a SINE WAVE:

The frequency domain can be explored using the FFT function. Let’s start over, just to make sure:

AGILENT 33120A:
SINE WAVE
1 kHz
2 Vp-p

AGILENT 54600B SCOPE:
AUTOSCALE
+/- key.
Function 2 ON.
Toggle the OPERATION softkey to get FFT.
FFT MENU
AUTOSCALE FFT
WINDOW: Toggle if necessary, to see HANNING
The display should look like FIG 6:

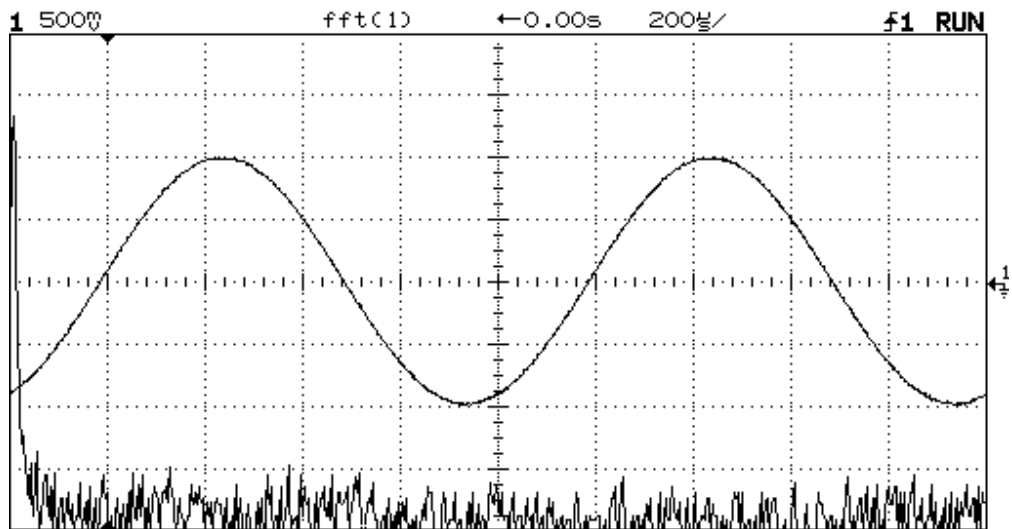


FIG 7: FFT of a sine wave

FFT of a SINE WAVE (cont’d):

We can improve the resolution on the fundamental by slowing down the horizontal sweep speed, essentially giving the algorithm many more cycles to work with:

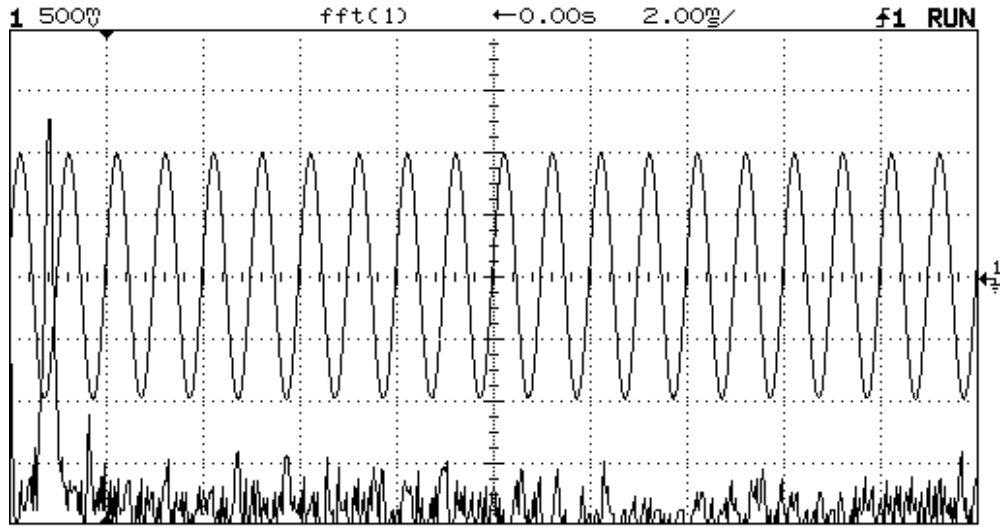


FIG 8: Higher resolution.

It's not always convenient to see both the time and frequency displays at the same time, so we can turn off the time display: Hit the CH 1 button twice. Then hit: CURSORS; FIND PEAKS, and the scope will automatically measure the peaks:

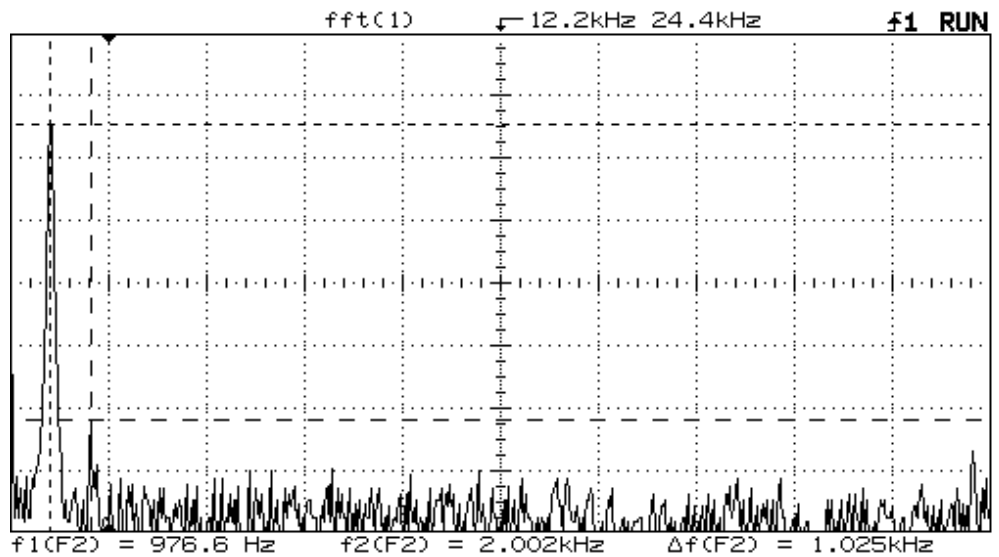


FIG 9: Find Peaks

The signal f1 should show 1kHz, but scope's counter resolution is not as good as that of the function generator. You can improve the frequency resolution by slowing down the horizontal sweep speed even further, but be careful not to go too far. Remember, the FFT is performed on EXACTLY what the time waveform depicts on the screen. To see the time waveform again, just toggle CH 1.

That concludes PART I of **Math Functions on a Scope**. MLF 5/97